

**DEPARTMENT OF NUCLEAR ENERGY**

The activity of the Department was focused on selected topics in nuclear fission reactor science and engineering. Present and future industry competitiveness, economic prosperity and living standards within the world are strongly dependent on maintaining the availability of energy at reasonable prices and with security of supply. Also, protection of man and the environment from harmful effects of all uses of energy is an important element of the quality of life especially in Europe. It is unrealistic to assume that the technology for renewable (hydro, wind, solar and biomass) sources of energy available within a 20-30 year perspective could provide the production capacity to replace present use of nuclear power and at the same time substantially reduce the use of fossil fuels, especially when considering that energy demand in industrialized countries can be expected to continue to increase even within a framework of overall energy conservation and continued improvement of efficiency in energy usage.

In the area of nuclear fission, we continue efforts to maintain and develop the competence needed to ensure the safety of existing and future reactors and other nuclear installations. In addition attention is given to explore the potential for improving present fission technology from a sustainable development point of view. The focus is on advanced modelling of improved reactor and fuel cycle concepts, including supporting experimental research, with a view to improving the utilisation of the inherent energy content of uranium and other nuclear fuels, whilst at the same time reducing the amount of long-lived radioactive waste produced. A common scientific understanding of the frequently used concept of "reasonable assurance of safety" for the long-term, post-closure phase of repositories for spent fuel and high-level waste is developed in order to ensure reasonably equivalent legal interpretations in environmental impact assessment and licensing procedures. Also, research is carried out on characterising and assessing the impact of disposal on

the environment with an aim to propose standards for protection.

Therefore, the following domains are subject of investigation:

- Analysis of new concepts of nuclear reactors with improved utilisation of nuclear fuel and reduced amount of long-lived waste, with higher efficiency in electricity production and energy supply for chemical processes.
- Problems of spent fuel management: technical and safety problems of spent fuel storage, inspection of spent fuel condition after long time storage in water pool, environmental impact of spent fuel storage, disposal and economy of spent fuel management
- Analysis of safety properties and characteristic of nuclear power plants with WWER reactors operating in the neighbouring countries.
- Analysis of the interaction of high energy hadrons with nucleus, especially looking for reaction with positive energy balance.
- Investigation of the nuclear processes applied for transmutation of transuranium isotopes and fission product. These studies are pursued at the collaboration of Joint Institute of Nuclear Research.
- Investigation of PWR fuel behaviour with high burn up and high temperature.
- Numerical methods for solving large linear system of equations, appearing in nuclear reactor physics as well as in many other areas of science and engineering,
- Study of effectiveness of the volatile pollutants removal from flue gas using electron beam technique. These studies are pursued in collaboration of Institute of Nuclear Chemistry and Technique.

## EUROPEAN ELECTRICITY SUPPLY SYSTEM OF XXI CENTURY AND BALANCE OF ITS ENERGY SOURCES

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Electricity is of paramount importance for modern industrialized economy. Serious breakdowns or interruptions in the supply of electric power would lead immediately to an almost complete breakdown of public and private life in developed countries. This is a fundamental concern that must be taken properly into account in any prospective scenario. Substantial economic growth and industrialization are expected around the world during the 21 st century, thus continuing the trend of the 20 th century. As a result, average living standards will rise, leading to a strong increase in energy – and especially electric energy - demand, even if the efficiency of energy conversion increases and the potential of energy conservation is fully used.

In the first few decades of 21 st century most of electric power stations operated until now will be shut down due to reaching the end of their service life time. In this area the new power stations will be designed to balance electricity demand due to closed stations and demand rise. It is estimated, that in period 2000 ÷ 2020 new power plants in the world will reach power level about 3 000 GW(e) [1]. In Europe new power stations in this period will reach the level of 500 GW(e). The estimated electric energy demand will reach level of 4 000 TWh per year. For generation of such amount of energy it is necessary to use about 860 Mtoe of primary energy. In 2000 the production of primary energy in Europe was 496 Mtoe and will decrease year by year [3].

The new power stations will be operated for the next 40 ÷ 50 years. The forecast of need for fuel for new electric power plants should take into account its availability, price and environmental impact. These factors play very important role, especially in Europe. The European countries have - with the exception of coal - no long-term fossil fuel resources. As stated in the EU Commission's Green Paper [2], the dependence on energy imports will drastically increase in the European Union within the next few decades, especially on oil from the Middle East and gas from Russia. At the same time, competition for oil and gas resources will dramatically increase due to the growing energy demand in the developing countries, which in turn may lead to a further rise in energy prices. It was this argument that led to the decisions of France, Japan and other countries during the oil crises of the 1970s to intensify their nuclear energy supply in order to be more independent of energy imports. In addition to competition on a global level the environmental

problems due to the burning of fossil fuels will still represent a serious threat in the 21 st century, especially for the European countries, as one of the most developed world regions and one of the largest consumers.

In the set of analyses it is shown that is unrealistic to assume that the technology for renewables (hydro, wind, solar and biomass) available within a 20-30 year perspective could provide the production capacity to substantially reduce the use of fossil fuels and replace planned shutdowns of nuclear plants. It will be especially the case, when electricity demand in European countries can be expected to continue to increase in the future, even within a framework of overall energy conservation and continued improvement of efficiency in energy usage.

Therefore, the planned resignation from nuclear energy in Europe is unrealistic [3]. It is expected, that new nuclear power plants will be designed to replace the shut down nuclear stations and for covering new demand of electric energy. Also, the new nuclear plants will be put into operation in other world regions, especially in North America.

In Poland, after accession to EU a growth of electric power demand will be expected. The economy of Poland can not be based on unparalleled energy saving devices. Therefore, the new plants with power of 25 ÷ 30 GW should be designed to be put into operation up to 2040 year. The primary energy for supplying such plants will reach the actual level of production of hard coal and lignite in Poland: 64 Mtoe. Therefore, a large amount of raw materials for electricity generation will be imported to Poland. Taking into account the environmental impact and EU Directive 2001/80/EC (which strongly limited the emissions of harmful substances from combustion plants) the nuclear power in Poland has good perspective.

### References

- [1] World Energy Outlook 2000, International Energy Agency, Paris 2002.
- [2] Green Paper of the European Commission: Towards a European Strategy for the Security of Energy Supply; Document COM (2000) 769 of 29 November 2000.
- [3] S. Chwaszczewski,; *European Electricity Supply System of XXI Century and Balance of its Energy Sources*. Polityka Energetyczna.

## OVERVIEW OF EMERGENCIES AT RESEARCH REACTORS

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The report was prepared as an invited lecture for the IAEA Workshop on Research Reactor Safety held in Warsaw, Poland, 22-26 Sept. 2003. It presented the principles of defence in depth philosophy applied to research reactors, the review of the main types of accidents possible in research reactors, lessons learned from these accidents and conclusions concerning possible hazards to the personnel and environment.

### **1. Safety of research reactors - result of safety philosophy and lessons learned in accidents**

The system of barriers in research reactors was compared to that in power reactors and the differences were highlighted, in particular the significant role of water in mitigation of fission product releases. Research reactors are as rule designed so that the reactor core is situated in a water pool, and in the case of any accident involving fuel damage the fission products from the core are scrubbed in the water pool before they get released to the atmosphere. High effectiveness of this scrubbing process for retention of iodine and other fission products was demonstrated in many experiments and those few accidents that happened in water pool reactors. As the accumulated thermal energy in the reactor core is small compared to that in power reactors, the retention of fission products in water and the system of underpressures maintained in the reactor by the ventilation system make it possible to mitigate effectively possible releases even though the leaktightness of the reactor buildings in research reactors is much inferior to that in nuclear power plants.

The review of reactivity initiated accidents and destructive tests of reactor fuel conducted in reactors dedicated to such studies showed how the errors and faults of reactor equipment characteristic for early years of research reactor development have been effectively removed, and how safety rules being in force today have been developed. The accidents due to loss of flow were also characteristic for the early years of research reactor technology, with no such accidents over the last quarter of the century. This is evidently the result of lessons learned in the accidents in the 1960'ies, when flow blockages across the cores of MTR type reactors occurred in several research centres.

The accidents that still happen from time to time are connected with the operations with fuel elements and with irradiated containers. However, the hazards involved have been also significantly decreased. In an early accident during fuel manipulation in NRU reactor the lack of proper cooling resulted in fuel burning and severe contamination of a large part of reactor hall, with cleanup by 600 men and reactor shutdown for 2 months. In the case of comparatively recent fuel handling accident in EVA reactor, where the spent fuel element was broken into two parts during transport from the core

to the spent fuel pool, the consequences were limited to negligible fission product releases, which did not cause any radiological hazards nor any measurable radiation doses to personnel or population. Within 3 days of the accident the reactor cooling system was put into operation, and two days later the reactor was started again.

Similarly, the incidents that happen during irradiation or handling of radioactive isotopes have very limited radiological consequences.

The comprehensive review and analysis of accidents in research reactors presented in the report were the basis for the following conclusions:

### **2. Typical phenomena preceding incidents**

The review of the accidents shows that in most cases the lack of safety culture, which eventually culminated in reactor incidents could be observed much before the incident itself. A classical example is SL-1 accident, but the predecessors to the incidents were usually observed also in minor cases. E.g. in the case of fuel break in Eva reactor the previous mechanical failures of the handling device had been unreported since they did not result in radiological consequences. It was only the final accident of fuel break that gave impetus to redesigning the gripping device, whose failure was the direct cause of the accident.

In many cases the faulty indications of various measuring devices had been tolerated before the incidents came. Then, during the incidents, the indications of trouble were considered not to be trustworthy, just because previously the device had been faulty. Such were the cases of reactivity measurements and flow measurements in the incidents with loss of flow described in the paper. More specifically, the loss of flow accidents were preceded by indications of:

- Reactor power decrease (due to local boiling)
- Reactor power reading fluctuations.
- Increased noise in readings of reactor parameters (due to boiling) such as servo error signal, count rate meter.
- Increased activity read by N-16 system ( due to the beginning of fuel damage).
- Radiation alarms (due to fission products releases), usually involving immediate reactor scram.

### **3. Hazards to personnel**

Significant hazards to personnel were created by large reactivity initiated accidents (RIA). In other cases of incidents and accidents the hazards to personnel were usually minor and in most cases no significant doses were obtained. It is worth stressing that the only fatalities were due to RIAs. RIA occurred both in

critical assemblies (Boris Kidric Institute – 6 people irradiated to 6.8 Sv, 1 fatality) and RA-2 (1 fatality), and in large experimental or research reactor (NRX – no fatalities, doses below 0.04 Sv), (SL-1 - radiation field above 5 Sv/h, 3 fatalities, rescue team up to 0.27 Sv).

The hazards due to loss of flow accidents could be kept under control by accident management measures, such as reactor hall evacuation, directing water to hold/up tank, primary coolant purification, and switching on emergency air ventilation filters. The retention of fission products in water pools was a significant factor in these accidents. A good example of the effectiveness of radiation protection was the fact, that after the accident in ETR, in which 6 fuel elements were burned out and 13 g of enriched uranium lost to the primary coolant, the total collective dose to the personnel was only 0.12 man/Sv. The role of the shielding provided by reactor building was shown in Siloe accident, in which the dose rate inside the building on the bridge was 2.5 Sv/h, but outside the shielding wall in the control room only 0.25 mSv/h.

The dose rates around primary piping after several loss of flow accidents were significant, and reached 0.1 Sv/h at demineralizer anion column in case of ORR accident, (partial melting of 1 fuel plate, 3 to 5 g of fuel melted), 0.35 Sv/h at heat exchangers in the case of Siloe accident (18 grams of enriched uranium melted and transferred to the primary coolant), 1 Sv/h at the heat exchangers after BR-2 accident with uncovering of 100 cm<sup>2</sup> of fuel surface.

In the case of incidents during isotope irradiation and fuel handling the doses to personnel are small, although the decontamination work necessary to remove radioisotopes from the walls can be time consuming. The only event with significant personnel irradiation was the NRU accident, in which a loss of coolant occurred to the hot fuel element being removed from the reactor (radiation field in excess of 10 Sv/h, the highest personnel exposure 0.19 Sv to one man). This stresses the necessity of careful planning of fuel transfer operations, especially in case of hot fuel, requiring effective water cooling.

DATE	REACTOR	POWER, MW (OR ENERGY MW-S)	COOLANT/MOD ERATOR	RELEASE OF FISSION PRODUCTS, Bq		
				IODINE	NOBLE GASES	SOLID FISSION PRODUCTS
12/12/52	NRX	30	H2O/D2O	NEGLIGIBLE	3.7 E14	NEGLIGIBLE
9/10/57	WIND SCALE	-	AIR/GRAPHITE	8 E14	1.2 E16	4.8 E14
15/10/58	KIDRIC INSTITUTE	0	D2O/D2O	-	-	-
3/4/60	WRT	60	H2O/H2O	0	9.6 E12 (EARLY)	0
3/1/61	SL-1	3 MW, 155 MW-S	H2O/H2O	2.6 E12	3.7 E14	2.2 E10
12/12/61	ETR	170	H2O/H2O	0	2.2E11	0
1/7/63	ORR	24	H2O/H2O	0.5 E10	1.8 E13	0
1/4/64 TEST	SNAPTRAN 2/10A-3	BURST, CUM. 45 MW-S	ZRH, REFLECTOR H2O	0%	3-4%	0%
11/1/66 TEST	SNAPTRAN 2	BURST, CUM. 54 MW-S	ZRH, REFLECTOR BE	70%	75%	21%
21/1/69	LUCENS	30	CO2/D2O	0	5.5 E10	0
28/3/79	TMI-2	2700	H2O/H2O	6.7E11	4.8 E17	0
23/9/83	RA-2*	0	H2O	-	-	-
26/94/86	CHERNO- BYL	3200	H2O/GRAPHITE	2.7 E17	4 E19	E18

\*Doses to the operator: 21 Gy due to gamma And 22 Gy due to neutron irradiation

#### 4. Review of fission product off-site releases in incidents with core damage in research, prototype and commercial nuclear reactors

As seen from the table, the off-site releases in accidents with research reactors were many orders of magnitude smaller than the releases from accidents in NPPs. In the worst research reactor accident, which occurred in SL-1 and caused death of all operators, the releases of noble gases and iodine were 100 000 time

smaller than in Chernobyl accident, and the releases of solid fission products were 50 million times smaller. Since it is the releases of Cs and other solid fission products which involve most serious long term radiological consequences it can be stated, that the hazards to environment due to research reactor accidents are negligible in comparison with the potential hazards presented by accidents in NPPs.

This is due not only to the smaller power of research reactors, but also to the important role of the water pools in retention of iodine and solid fission products. This is shown in particular by the radiological effects of two SNAPTRAN experiments, one of which was conducted in a water tank and the other in the air environment. The result was that there was practically no iodine release in the first case, while in the second the release of iodine reached 70% of core inventory. Another factor that contributes to the retention of fission product is the comparatively low and fast decreasing temperature in the primary coolant system and in the

reactor building. The general observation is that while the accidents in research reactors can be dangerous to the operating staff, they are of negligible importance as far as the hazards for the population are concerned.

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- [1] A. Strupczewski: *Overview of Emergencies at Research Reactors*, invited lecture at the IAEA Workshop on Research Reactor Safety, Warsaw, Poland, 22-26 Sept. 2003.

### METHODOLOGY OF COMPARISONS OF ECOLOGICAL ASPECTS OF ENERGY SOURCES

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The paper was prepared as an invited lecture for the International Conference "Electric Power Supply Strategy in the 21<sup>st</sup> Century", held in Warsaw on 1-3 October 2003. It stressed the need for Life Cycle Analysis that does not neglect any stages in the energy production cycle, the choice of representative technologies for comparisons based on historical events, the full coverage of the spectrum of emissions and their impacts. The influence of long term effects and of relation of man made pollution to the background level was shown and the methodological aspects of typical arguments against nuclear power were discussed.

Non quantifiable elements of comparative assessment were reviewed including aversion to large accidents, global warming and waste management considerations.

When we consider all stages of energy production according to Life Cycle Analysis principles it turns out that nuclear power is among those energy sources that use the smallest amounts of structural materials, human work and energy per unit power produced. The renewable energy sources, which do not produce any pollution during plant operation, are a significant burden for the environment because very large amounts of materials and energy are needed for their construction, and this involves large emissions of air pollutants. The paper demonstrates that neglecting environment and health burdens of some stages of energy production cycle leads to biased and misleading results.

Technological progress reduces pollution and accident hazards involved in energy production. Both historical experience, showing great changes in e.g. dam break prevention, and the recent studies in which the real technology is considered for comparisons of future power plants, indicate that the evaluation of future power plants, nuclear, conventional, or using renewable energy sources, should be based on the best actual technology available on the market. This is important for nuclear power, because Chernobyl accident cannot and should not be the basis for evaluation of potential accident hazards of nuclear power plants.

In normal operation nuclear power plants release so small quantities of fission products, that the resulting additional doses are only very small fractions of natural background radiation, much smaller than the natural variations between countries or regions within a country. In the case of air pollution however, the doses of pollutants due to human activities are many times higher than the background concentrations in unpolluted areas.

A discussion is also presented of the claims of nuclear power opponents, and the results of large studies conducted for NPPs in the US, for Sellafield reprocessing plant in UK and for La Hague reprocessing plant in France are presented. In all of these studies no detrimental effect of nuclear facilities on human health was found. For example, in the case of La Hague, where a multi-year study was prompted by the claims that 2 additional leukemia cases are due to nuclear pollution, the study showed that the maximum possible number of leukemia cases over the whole period of plant operation (20 years) and in the whole population exposed could be 0.0014 cases. Similarly, the opponents of nuclear power present estimates of nuclear accident hazards which are many, many times higher than any reasonable estimates, e.g. the statements of Hochmeyer are about 80 000 times too high in comparison with the objective study made by the most authoritative Paul Scherrer Institute in Switzerland, developing unbiased estimates for Swiss government.

Consideration of other elements, like global warming and waste management issues also indicates that nuclear power is one of the best energy options open to the man today.

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## THE ENERGETIC BALANCE IN DIFFERENT STAGES OF THE EXOERGIC NUCLEAR REACTIONS IN HADRON-NUCLEUS AND NUCLEUS-NUCLEUS COLLISIONS

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### 1. Introduction

The exoergic nuclear reactions are the reactions, during which energy is released. In general case there can be the fusion reactions (synthesis) of light nuclei or fission reactions (fragmentation) of heavy nuclei. The energy of reaction comes from change of the mass of nucleus into energy.

There are a lot of nuclear reactions during which, positive energy during which is released. In practice however fission of heavy nuclei in practice was used (uranium and plutonium). In this work we will mainly describe the fission and fragmentation reactions. We will introduce the most important profiles of process of fission, which decide about quantity and intensity of

released energy. The intensity of fission reaction depends on total cross section, the threshold energy, distribution of masses and charges of fragments of reaction as well as binding energy of nuclei (before reaction) and of the fragments of nuclei (after reaction). The energy of reaction is released as kinetic energy of light and heavy fragments of reaction.

### 2. The fission and fragmentation reaction

Excitation of nucleus is a basic physical phenomenon bringing nucleus to fission. This state of nucleus can be to induced by quantum gamma, meson  $\pi$ , neutron, proton, deuteron,  $\pi$  particle or other nucleus.

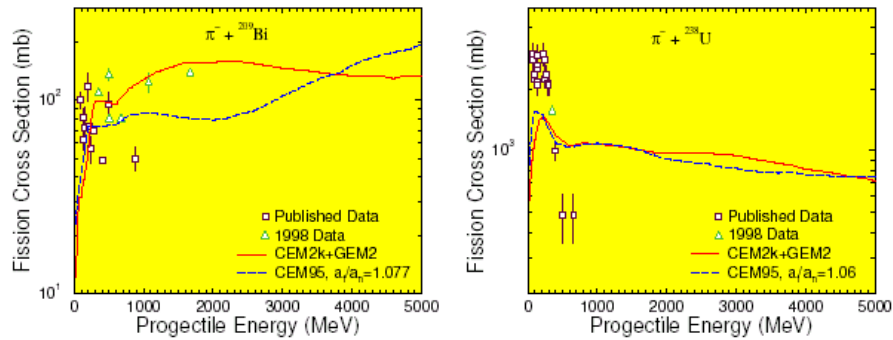


Fig. 1. Comparison of total cross section of fission nucleus  $^{209}\text{Bi}$  and  $^{238}\text{U}$  induced  $\pi$ -meson as function of energy of hadron. Reactions  $^{209}\text{Bi}(\pi^-,f)$  and  $^{238}\text{U}(\pi^-,f)$ . Experimental theoretical data. The figure comes from the work [2].

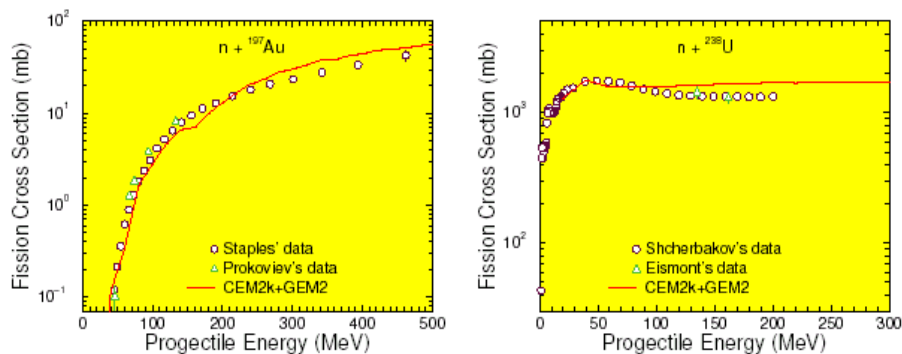


Fig. 2. Comparison of total cross section of fission nucleus  $^{197}\text{Au}$  and  $^{238}\text{U}$  induced neutron as function of energy of hadron. Reactions  $^{197}\text{Au}(n,f)$  and  $^{238}\text{U}(n,f)$ . Experimental and theoretical data. The figure comes from the work [2].

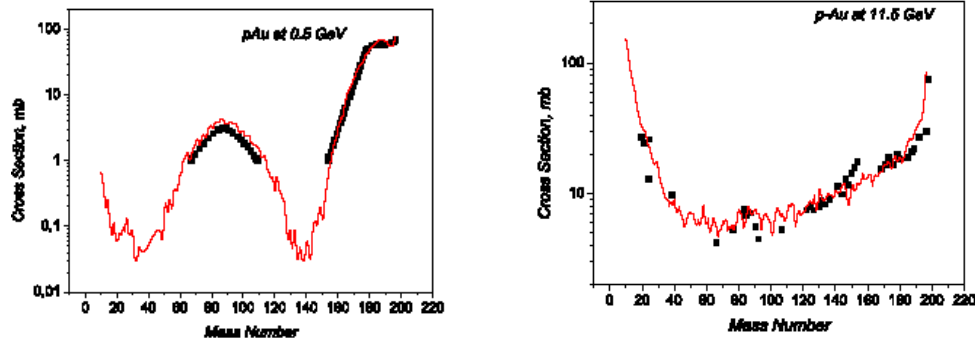


Fig. 3. Comparison of distribution of masses of fragments reaction  $p + {}^{197}\text{Au}$  induced hadron with energy 0.5 GeV. and energy 11.5 GeV. Experimental and theoretical data. The figure comes from the work [7].

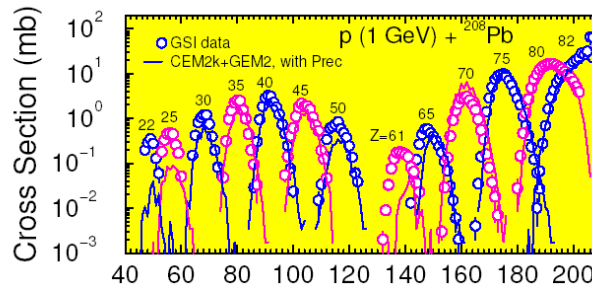


Fig. 4. Distribution cross section of production 13 isotopes with charge  $Z = 22, 25, 30, 35, 40, 45, 50, 61, 65, 70, 75, 80, 82$ . Reaction  ${}^{208}\text{Pb}(p, f)$  with the hadron energy 1 GeV. The figure comes from the work [4].

### 3. The energetic balance of nuclear reaction

To calculate the energetic balance of nuclear reaction was one should know distribution masses and electric charges of all fragments formed in nuclear reaction. The energetic balance is the difference between total mass (resting energy) of elements of reaction before and after reaction

$$E_A^s = Zm_p c^2 + (A-Z)m_n c^2 - \Delta W(A, Z) + m_h c^2$$

$$E_{A'}^s = \sum_i (Z_i' m_p c^2 + (A_i' - Z_i') m_n c^2 - \Delta W(A_i', Z_i')) \quad (1)$$

$$+ n_p m_p c^2 + n_n m_n c^2 + m_h c^2$$

where  $E_A^s$  and  $E_{A'}^s$  -means total mass (resting masses) before and after reaction,  $A_i', Z_i', n_p, n_n$  - means the mass number and charge number the complex products of reaction, number of free protons and neutrons,  $\Delta W$  - the binding energy of nucleus.

The difference between initially resting mass and final mass is the energy of reaction. For exoergic reaction the energy of reaction Q is positive:

$$Q(A, Z, A_i', Z_i' n_n, n_p) = \sum_i \Delta W(A_i', Z_i') - \Delta W(A, Z) \quad (2)$$

Numbers  $A_i', Z_i' n_n, n_p$  are called the channel of reaction.

According to experimental data (chapter 2) the probability of reaction through given channel is the complex function of energy of hadron as well as of kind nucleus and hadron.

Energy reaction Q is different for different channels of reaction. For different channels of reaction it can be positive or negative. To calculate the energy of reaction Q it is necessary to calculate the binding energy of individual (complex) fragments of reaction. It was calculated using well-known Wassaker formula.

If we mark the probability of fragmentation of nucleus through channel by P then the expected value of reaction energy will be calculated with formula:

$$\langle Q(A, Z, A_i', Z_i' n_n, n_p) \rangle = P(A, Z, A_i', Z_i' n_n, n_p) Q(A, Z, A_i', Z_i' n_n, n_p) \quad (3)$$

Calculations of reaction energy is a complex problem because we should know all channels of reaction and corresponding cross section. The average energy released during reaction can be simply calculated using a semi-empirical method based on the experimental data for individual channels of reaction. The cross section of individual channels could be calculated using existing programs (CEM2k + GEM2, LAHET).

### 4. Conclusion and remarks

The production of fission reaction energy of the any heavy nucleus is a very complex process. Now in practice the fission energy is produced only with reaction  ${}^{235}\text{U}(n, f)$ ,  ${}^{238}\text{U}(n, f)$  and  ${}^{239}\text{Pu}(n, f)$ . This is due to the ease of fission of these elements with large cross sections and no threshold fission energy. Using of different elements is not so simple due to smaller cross sections and considerably larger threshold energy, although this is possible with present state of physical

knowledge and technique. The problem depends on finding of suitably cheap method of releasing of energy existing in heavy nuclei.

One should underline that in this work the results of basic investigations our fusion (synthesis) of light nuclei were skipped as well as the results of construction of reactors using the fusion of light nuclei.

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## THE INVESTIGATION OF TRANSMUTATION PROCESSES IN COLLISIONS OF THE HIGH ENERGY NEUTRON BEAM WITH NUCLEI OF BISMUTH IN EXPERIMENT "ENERGY PLUS TRANSMUTATION "

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### Introduction

Nuclear transmutation in collisions of the high energy neutron beam with nuclei of different elements was the basic part of the international experiment "Energy plus Transmutation" executed in JINR in Dubna. This work deals with the quantitative description of products the transmutation produced from  $^{209}\text{Bi}$  (bismuth) targets. The experiment took place in December 2001.

### 1. Experiment description

The basic part of the experimental model is an assembly U/Pb (fig.1) built up of lead target and natural uranium cylinders. The assembly is divided into four identical longitudinal sections between which foil with activation and track detectors are placed. These foils are placed also at front and back of the assembly U/Pb. During irradiation in proton beam the assembly U/Pb is placed in special wooden chests (Fig.2.) filled with polystyrene. The time of irradiation of the assembly in proton beam was equal to 43800 sec. The integral of

intensity of proton beam during the whole experiment was equal to  $10^{13}$  protons.

The samples of bismuth were placed on every foil (near the target) in different distance from the target axis. Thanks to this we can find the dependence of distribution of produced isotopes from longitudinal and radial distance.

The experiments consisted of two parts:

- The irradiation of the assembly U/Pb (Fig. 1.) with the beam of high energy protons (1GeV) from synchrophasotron.
- The measurement of the gamma spectra of activation detectors situated in different places of the assembly U/Pb.

The measurements of gamma spectrum were performed in three part (series of measurements) to detect isotopes with different half life periods.

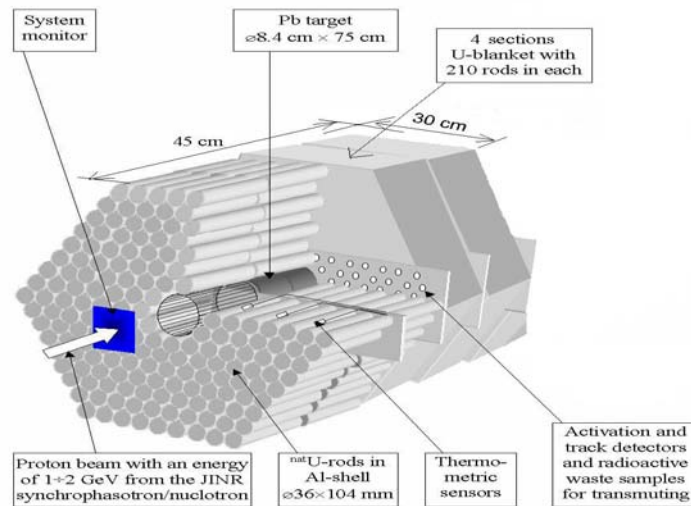


Fig. 1. Experimental assembly Pb/U (four section model).

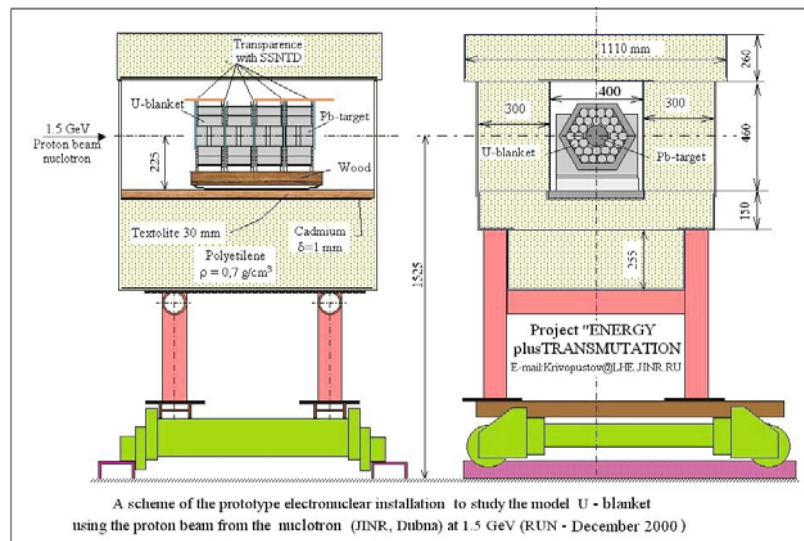


Fig. 2. A scheme of the "Energy plus Transmutation" installation.

At the beginning isotopes with the shortest lifetime then those with medium and in the end those with the longest lifetime were measure. The measurements of first series lasted 1800 sec., second

-3600 sec. and third - 86000 sec. During these measurements a several standard target and background measurements were performed to calibrate detectors. Every measurement was recorded in a separate file in format \*.CHN.

## 2. Analysis and calibration of results

The first stage of analysis was executed by the computer program DEIMOS. This programme was used to find the energetic calibrations as well as spectra lines of individual isotopes. This programme finds fits to given line shape and calculates its base parameters (surface, half width and measurement errors).

The second stage of analysis consisted in recognizing and identification of spectra lines coming from disintegration of different isotopes. It was using 'hand procedure' of recognizing isotopes to identify the maximum number of isotopes (presence of some isotopes was not trivial and easy to predict). For identification of spectra lines the tables of isotopes on CD-Re from Lawrence Berkeley National Laboratory were used.

In the third stage of analysis a calibration was performed by mass of samples and the time of measurement. We considered the duration of experiment time (irradiation in proton beam), the time that passed since irradiation to gamma spectrum measurement as well as duration of gamma spectrum measurement. The calibration formula is presented below:

$$N_o = \frac{N_1}{m} * \frac{\left( \frac{\ln(2)}{h/2} * T_{ira} \right)}{\left( 1 - \exp \left[ - \frac{\ln(2)}{h/2} * T_{ira} \right] \right)} * \left( \exp \left[ \frac{\ln(2)}{h/2} * T_+ \right] \right) * \frac{\left( \frac{\ln(2)}{h/2} * T_{real} \right)}{\left( 1 - \exp \left[ - \frac{\ln(2)}{h/2} * T_{real} \right] \right)}$$

where No- the actual the size of line ( the activity)  
 N1- the experimental size of line  
 h /2- the time of partial disintegration  
 T<sub>ira</sub> the time of duration of experiment (the irradiation)  
 T<sub>+</sub> the time between end of experiment and beginning of measurement  
 T<sub>real</sub> the time of measurement  
 m mass sample(target) in [grams]

The fourth stage of analysis is calculation of efficiency of production of individual isotopes in [gram/proton] or [Bq/proton]. In analysis one should calculate corrections on detector efficiency as well as cascade effect. Moreover one should perform differential analysis of line to identify results with large statistical error.

**3. Presentation of results**

During the analysis of samples of <sup>209</sup>Bi the following isotopes were identified <sup>206</sup>Bi, <sup>204</sup>Bi, <sup>203</sup>Bi, <sup>202</sup>Bi, <sup>201</sup>Bi and <sup>200</sup>Bi, as well as <sup>203</sup>Pb. The results are presented in table 1.

Table 1.

Probe Bi209				4	Bi203 11.76h	42336	186.5 722.4			
Nr:	Name of isotopes	half live (s)	Energy (keV)							
1	Bi207 31.55y	994960800	569.702				820.3			
			1063.662				825.2			
2	Bi206 6.243d	539395,2	183.977				847.3			
			343.45				896.9			
			398.00				1033.8			
			497.06				1253.7			
			516.18				1506.7			
			537.45				1536.4			
			620.48				1679.5			
			657.16				1748.5			
			803.10				1847.4			
			881.01				1893.0			
			895.12				2011.4			
			1018.63				2101.4			
1098.325	2191.4									
1595.27	2281.4									
1718.70	2371.4									
3	Bi204 11.22h	40392	216.01				5	Bi202 1.72h	6192	240.22
			219.38							346.47
			222.32							422.18
			374.72	578.56						
			440.59	657.49						
			501.94	927.29						
			670.75	954.45						
			791.19	960.67						
			718.70	6	Bi201 108m	6480	1245.47			
			895.12				171.7			
			911.78				629.1			
			918.15				936.2			
			984.02				1014.1			
							1325.2			
							419.7			
							462.34			
				7	Bi200 36.4or31m	2184	1026.5			
							279.1967			
				8	Pb203 51.873h	186742				

There is a possibility of the presence of isotope <sup>207</sup>Bi, but its identification is very unsure. The presence of other possible isotopes of bismuth could not be detected because they have a very long the half life time ( of about hundreds thousand years –<sup>210</sup>Bi, <sup>208</sup>Bi) or a very short half life time (of about minute or a second - Bi<sup>199</sup>).

Fig. 3. below shows the changes of the surface of line of isotope Bi<sup>204</sup> coming from isotopes Bi<sup>209</sup> as a function distance from beginning (along) axis of lead target. Fig.4 presents changes of the surface of line as a function of a distance from the axis of lead targets for isotope Bi<sup>204</sup>. The figures for produced isotopes of Bi<sup>206</sup>, Bi<sup>204</sup>, Bi<sup>203</sup>, Bi<sup>202</sup>, Bi<sup>201</sup>, Bi<sup>200</sup>, Pb<sup>203</sup> look similarly.

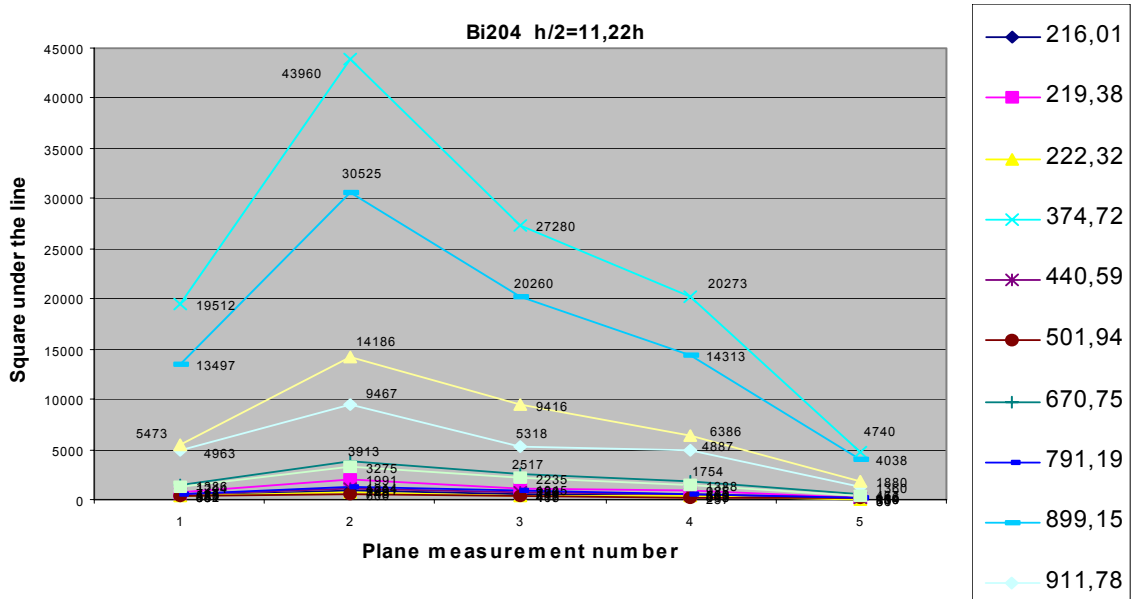


Fig. 3. The changes of the surface of line of isotope  $Bi^{204}$  come from isotopes  $Bi^{209}$  as a function distance from beginning (along) axis of lead target [number of foil]. The right table presents energy (keV) for individual lines.

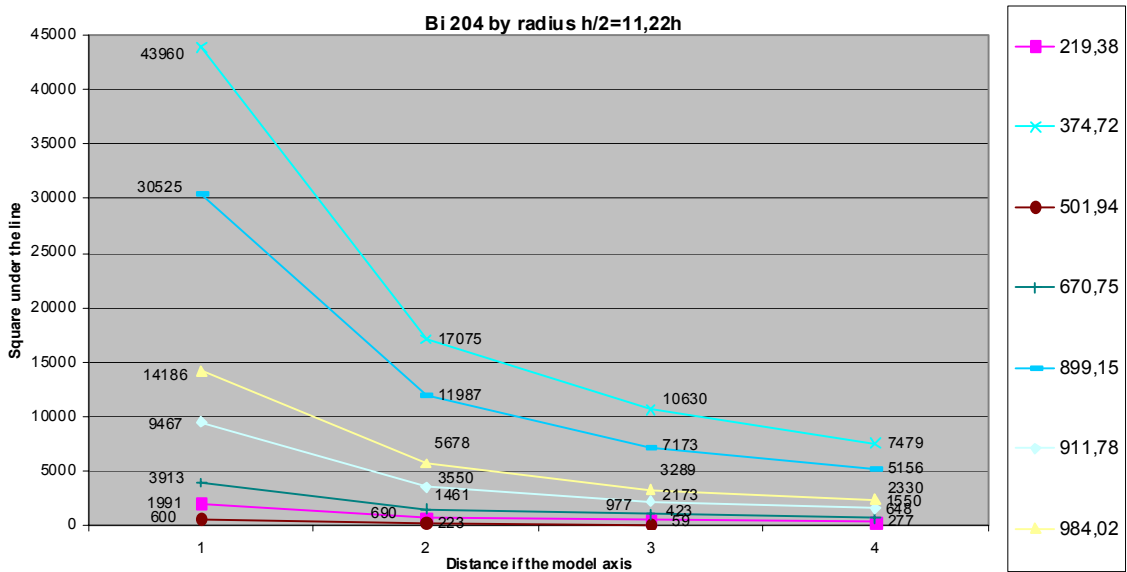


Fig. 4. The changes of the surface of line of isotope  $Bi^{204}$  come from isotopes  $Bi^{209}$  as a function of the distance [cm] from the lead target axis (point 1 – nearest to axis, point 4 – farthest). The right table presents energy (keV) for individual lines.

4. Conclusions and remarks

On basis of fig.3-4 the number of individual isotopes produced in the experiment in accordance with expectations is proportional to the neutron beam produced in assembly. For all produced isotopes clear maximum is in near second foil (fig.3). On the two final foils there is the clear decrease of content of produced isotopes. In this part of assembly the produced neutron beam is clearly lower.

On basis of fig.4 one may observe that the number of produced isotopes as a function of distance from lead target axis is sharply decreases. This due to decreasing

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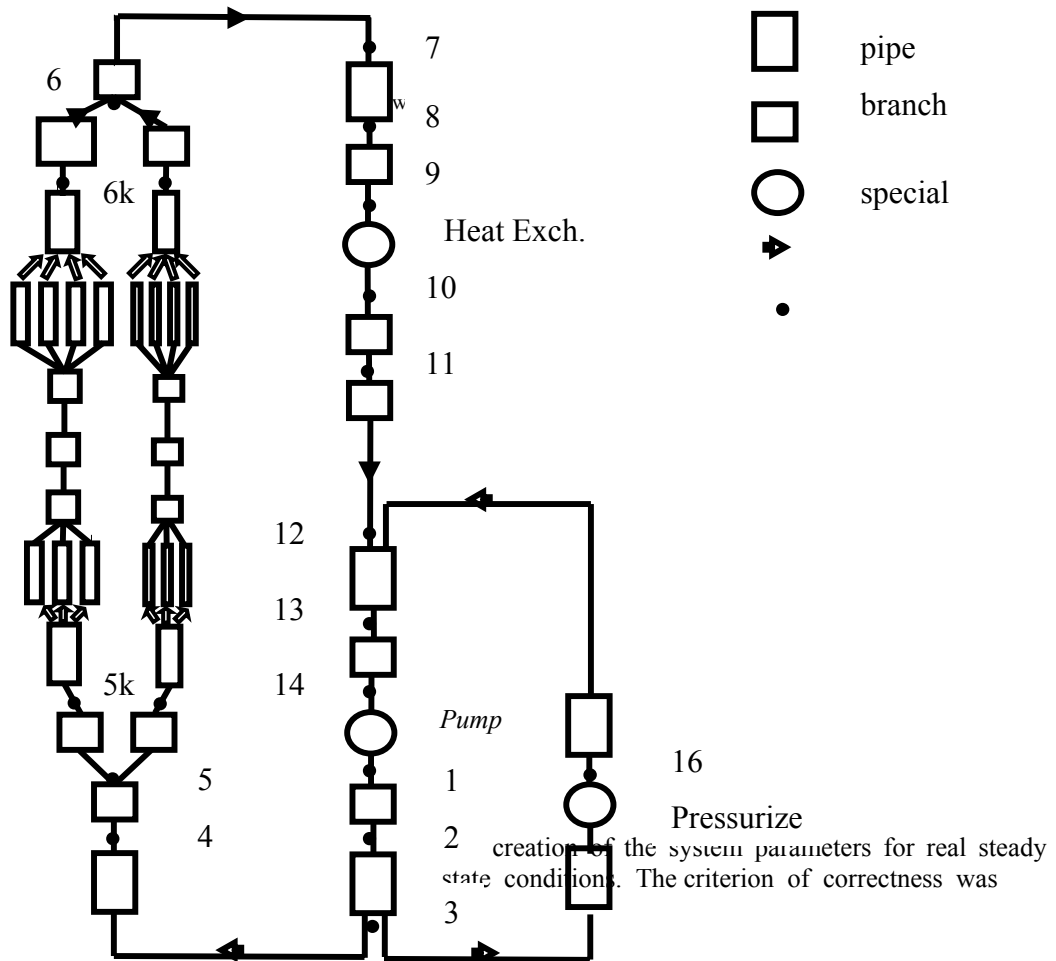
intensity and energy neutron beam.

**INPUT DATA PREPARATION OF THE MARIA REACTOR PRIMARY COOLING CIRCUIT MODEL IN RELAP5/MOD3 CALCULATIONS**

**Piotr Czerski, Jan Szczurek**  
*Institute of Atomic Energy*

The aim of the report [1] is to create input data model for RELAP5/MOD3 code, which describes the primary core cooling circuit of the MARIA research reactor. The input data model of the reactor cooling system includes the fuel assembly of the reactor core, inlet and outlet

collectors, the pipe system of the cold and hot legs, pumps, primary and secondary side of heat exchangers and the pressurizer with pressure stabilisation system. The nodalization scheme of the MARIA reactor core cooling system is shown in Fig. 1.



**Fig. 1. Primary circuit in MARIA reactor – nodalisation for RELAP calculations.**

Particular part of the system is represented as follows: pump system and pump collectors 13-14-1-2, main circulation pipelines 2-3-4; 7-8; 11-12-13, channel collectors 4-5; 6-7, pipes from fuel channels to channel collectors 5-5k , 6k-6, heat exchangers system and heat exchanger collectors 8-9-10-11, pressurizer system with its pipelines 3-16-12. The core model of the MARIA reactor developed in [2] is represented in Fig.1 by points 5k-6k.

The aim of the preliminary calculations was to examine the correctness of the RELAP5/MOD3 model

successfully fulfilled when values of pressure, temperatures, flow, heat generation and level of water in the pressurizer stabilised without significant fluctuations. Simulation of the steady state calculations has been performed for 1000 sec. Steady state calculations create starting point for transient simulations including accident scenarios, important for safety assessment.

**References**

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## INTRODUCTION TO PHYSICAL AND TECHNICAL ANALYSIS OF ACCELERATOR DRIVEN SYSTEM

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The main elements of the accelerator driven system (ADS) are discussed: spallation source, sub-critical reactor and accelerator. Describing the spallation source it is underlined that beside the well accepted theory of spallation, the spallation phenomena are not sufficiently investigated yet. Dealing with the sub-critical reactor as an energy amplifier of the primary spallation source a suggestion for the specific neutron spectrum is proposed to obtain optimal conditions for energy production, burn-up and transmutation. In the description devoted to the accelerators of charged particles, the two accelerator principles are presented on the examples of working accelerators: the linear accelerator – LINAC and cyclotron. Finally, there is presented the project of

accelerator driven system - MYRRHA for research and development worked out by the Belgian Nuclear Research Centre - SCK•CEN and the conception of nuclear power station of RBMK-1000 type with spallation source together with analysis of the balance of energy developed by the Joint Institute of Nuclear Research, Dubna, Russia [1].

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## XENON CONCENTRATION IN SUB-DIVIDED FUEL FOR VERY HIGH BURN-UP IN TERMS OF DEFECT TRAP MODEL

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Microstructure of the fuel following irradiation to high burn-up consists of several typical zones. Near the fuel surface is a zone with a typical thickness of 100 – 200  $\mu\text{m}$  where very small grains of less than 400 nm diameter exist. The experimental data show that the distribution of the sub-grain sizes appears to begin from 60 nm. The subdivision process is formed in the cold part of a fuel rod.

Since the mechanism leading to the formation of the nano structure in the  $\text{UO}_2$  fuel is not fully satisfactory understood at the present time, we focus on better understanding the conditions of formation of the nano structure in the  $\text{UO}_2$  fuel.

It is assumed that above a limiting value of burn-up a more intensive process of irradiation induced chemical interaction occurs. Significant part of fission gas product is thus expected to be chemically bound in the matrix of  $\text{UO}_2$  fuel. The fission gas atoms substituting for example uranium atoms in the crystallographic lattice can form weak facets. At certain saturation condition, division of the grains can occur at the weak facets and the increase in fission gas products release may be expected.

Simultaneously with the process of subdivision, the increase of local porosity and the decrease of local

density in function of burn-up occurs, what leads to the increase of total surface area.

Since the fission gas release from the  $\text{UO}_2$  grains during subdivision process is athermal it is inferred that the knock-out release process is significant.

It is assumed that the same processes exist for the transformed fuel as for the original fuel, with the difference that the total surface area is so big that the whole fuel can be treated as that affected by the knock-out process:

If some gas is trapped in the bubbles and it is experimentally proved that the formed gas bubbles are destroyed by the fission fragments, then some gas atoms have to be immobilised in the crystallographic lattice as gas atoms chemically bound with the fuel. So the same processes take place in the transformed fuel as in the original fuel, with the difference that the total surface area is so big that the whole fuel can be treated as that affected by the knock-out process described in the following defect trap model equations:

$$\frac{dN_g}{dt} = g_1 f - (g_2 + g_3) f N_g \quad (1)$$

$$\frac{dM}{dt} = \beta_1 f + \alpha_1 f M_r + g_3 f M_r - \alpha_2 M - g N_g M \quad (2)$$

$$\frac{dM_r}{dt} = gN_r M - g_2 f M_r - g_3 f M_r - \lambda M_r \quad (3)$$

$$\frac{dM_r}{dt} = \alpha_2 M - \alpha_1 f M_r - \lambda M_r \quad (4)$$

$$R = g_2 f M_r \quad (5)$$

$$\alpha_2 + gN_r = 1 \quad (6)$$

where

- $N_r$  - concentration of bubbles in the fuel,  
 $M$  - concentration of intermediate gas atoms,  
 $M_{tr}$  - concentration of gas atoms in the bubbles,  
 $M_r$  - concentration of gas atoms dissolved in the fuel,  
 $R$  - fission gas release rate,  
 $\lambda$  - decay constant of isotope  $i$ ,  
 $\beta_i$  - formation yield of the intermediate gas of isotope  $i$ ,  
 $f$  - fission rate,  
 $t$  - time,

$g, g_1, g_2, g_3, \alpha_1, \alpha_2$  - constants.

The above equations let to carry out the analysis of fission gas concentration change due to formation of nanostructures in  $UO_2$  fuel at high burn-ups in terms of total surface area change in function of burn-up and knock-out process [1].

## References

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## REMOVAL OF BENZENE FROM EXHAUST GASES USING ELECTRON BEAM FROM ACCELERATOR.

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Volatile organic compounds (VOCs) released into the atmosphere from various industrial processes cause a serious and large scale environmental contamination. Several kinds of VOCs are very harmful to human health and also produce toxic substances by photochemical oxidation in the atmosphere. New emission regulation established a list of potentially hazardous air pollutants with their emission limits. Many industrial facilities will need pollution abatement system for these pollutants in order to be in compliance. Many of the regulated pollutants are volatile organic compounds. VOCs have been conventionally removed using adsorption, incineration or condensation processes. Adsorption works well for low concentration of some VOCs. Incineration or condensation processes are typically used for high concentration of VOCs. If multiple pollutants are present in a gas stream at varying concentration multiple control technologies may be needed, adding to the cost and complexity of the process. It is highly desirable to have a control technology capable of removing multiple gas-phase pollutants, thereby reducing cleanup costs and process complexity. In the electron beam treatments, the energetic electrons produce gas-phase free radicals ( $O(^1D)$ ,  $O(^3P)$ ,  $OH^*$ ) and other active species, which

oxidize and decompose pollutants, such as benzene. The destruction of benzene was examined in the laboratory tests as a function of applied dose and benzene concentration.

### Experimental study

The model gases, air streams containing benzene were irradiated with electron beam in flow system (Fig. 1.). Gaseous benzene was generated by the bubbling of synthetic air, as a carrier gas, through liquid benzene. The temperature of the liquid benzene was set to 25°C with a water bath to keep its vapor pressure constant. The bubbled gas was diluted with the synthetic air from the second cylinder, as a dilution gas, in a gas mixer and introduced into the process vessel (reactor). The concentration of benzene in the sample gas was adjusted by changing the flow rates of bubbling gas and dilution gas as well as the temperature of liquid benzene. The concentration of 100 ppm benzene was obtained at the following set of parameters [1]:

- flow of carrier gas 1 l/h,
- flow of dilution gas 800 l/h,
- temperature of liquid benzene 22°C.

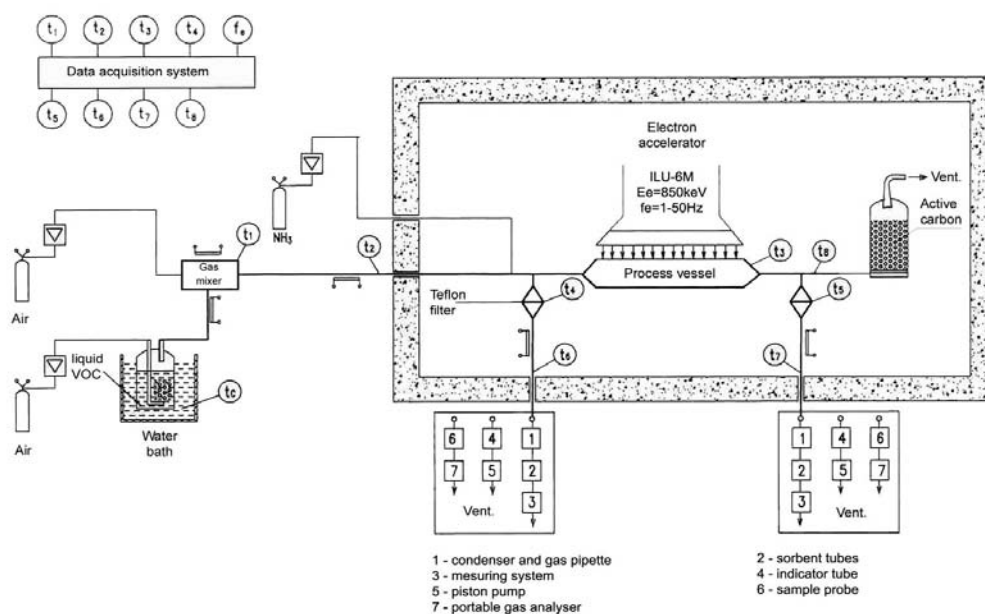


Fig. 1. Flow diagram of electron beam irradiation of benzene/air mixture.

The tubes through which gas mixture was transported to the reactor and the process vessel were kept at a temperature above boiling point of benzene to avoid condensation of benzene vapor.

A pulsed electron accelerator ILU-6M was used for electron beam treatment. The gas mixture was irradiated with 0.8 MeV vertical electron beam in a stainless steel flow type process vessel. The vessel has a cuboidal shape with total volume of 3 liters. An electron beam was introduced into the reactor through a 50  $\mu\text{m}$  thick titanium window. The dose deposited in the gas mixture was adjusted by changing repetition rate of electron pulses up to 50 Hz. The average dose in the reactor was calculated from the dose distribution determined using CTA film dosimeter (FTR-125, Fuji Photo Film Co.).

The gas mixture composition was determined at the inlet and outlet of the process vessel. These data were used for determination of the removal efficiency of benzene. Two independent heated sampling lines were installed. At the beginning of both lines there were heated pre-weighed teflon filter with 2.0  $\mu\text{m}$  pore size for collection of aerosols. After filtration, sample gas was transported to one of three different analytical setups. The main setup consisting of: a condenser, gas-pipette and gas adsorption tubes (two tubes of XAD-2 resin and one tube of activated carbon) was designed for gas analysis on GC/MS system. The second system consisting of specific indicator tube and piston pump was designed for determination of benzene concentration in the sample gas. The analyses of gaseous inorganic components of sample gas, like CO, NO, NO<sub>2</sub>, NO<sub>x</sub> and O<sub>2</sub>, were performed by flue gas analyzer (Lancom Series II, Land Combustion Ltd.).

## Results and discussion

The removal efficiency of benzene depends on the applied dose and its initial concentration. Fig. 2 shows experimental results.

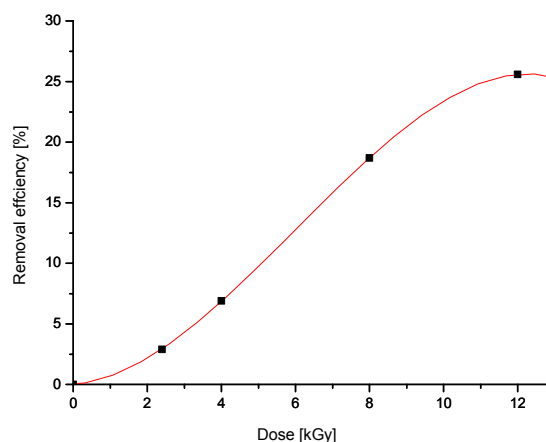


Fig. 2. Removal efficiency of benzene vapor as a function of irradiation dose for 105 ppm of benzene. Gas temperature in the process vessel was 100°C.

Decreased removal efficiencies were observed with increase of initial concentration of benzene. After electron beam irradiation, organic and inorganic gaseous products were formed. Table 1 presents identified products. CO, CO<sub>2</sub>, phenol, biphenyl and aerosols particles were found as products and were determined quantitatively. The experimental results showed that 30 % of decomposed benzene were converted into aerosols [2].

Table 1. Identified products of benzene decomposition.

Product	Identified technique
CO	Gas analyzer Land
CO <sub>2</sub>	GC/MS and GC-FID
Phenol	GC-FID
p-benzoquinone	GC/MS
Oxepin	GC/MS
Benzaldehyde	GC/MS
Nitrobenzene	GC/MS and GC-FID
Biphenyl	GC/MS

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## STUDY OF THE POSSIBILITY OF RADIOACTIVE WASTE TRANSMUTATION IN A DYNAMIC MODE OF OPERATION: PROPOSAL OF INVESTIGATION\*

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The problem of transmutation of radioactive waste (RW) coming in the main from nuclear plants as a spent fuel is nowadays of fundamental importance both from economic and ecological viewpoints. It has also a serious cognitive meaning although often underestimated by many physicists. Nevertheless, it is, along with the problem of efficiency and ecological safety, the basic one determining the present status quo and the future of nuclear power on the global scale (see, for example, review articles [1,2] and references quoted there). But the experiments performed up to day in different laboratories show that this problem represents on the practical level an exclusively difficult challenge: the transmutation appears to be surprisingly inefficient and enormously expensive, at least as conducted by the traditional way [2]. The point is when even a precisely isotopically selected sample of a given radioactive nuclide is irradiated in the neutron flux of  $\sim 10^{14} \text{ s}^{-1} \text{ cm}^{-2}$  intensity as the most efficient and usual, progressively other nuclides appear and during the first several years of irradiation the total radioactivity of the sample increases [3,4].

Much more complex situation occurs if the irradiated sample is not initially strictly determined isotopically but, for example, is simply taken as a piece of the spent reactor fuel.

So, we put forward a suggestion to use for transmutation a powdered or liquid target of RW circulating in the field of high energy and intensity

relativistic particles beams when from the exposed target are gradually extracted stable nuclides.

The stages of radioactive waste transmutation in a dynamic mode in the form of a principal scheme of numerical modeling are as follows:

1. The initial state of a sample of radioactive waste (RW) destined for transmutation should be determined isotopically, powdered and mixed:  $\alpha_i^{(0)}$  is a part of the  $i$ -th isotope (e.g.  $^{129}\text{I}$ ,  $^{237}\text{Np}$ ,  $^{239}\text{Pu}$  and all others constituents of the sample),  $V^{(0)}$  – is the initial volume of the sample as a parameter to be optimized.
2. The sample of RW is next exposed (in our approach by means of numerical modeling) to the (1) thermal-reactor, (2) fast-reactor and (3) spallation neutron beams of a flux  $\mathbf{f}_j$  during the time  $t_j$ , both values are the parameters to be optimized.
3. The dependence of parts  $\alpha_i(t|V)$  of transmuted isotopes should be evaluated and when the ratio  $\Delta\alpha_i/\alpha_i$  exceeds  $a_i$  (say,  $a_i \approx 0.1$ ) the RW sample is subjected to fast radiochemical (pyro-chemical) partitioning with the tolerance  $\delta\alpha_i$  for the  $i$ -th isotope. In this way the new values of  $\alpha_i^{(1)}$  and  $V^{(1)}$  appear and the processing of the RW sample starts again until, for example, the total radioactivity of the sample falls below some expected and practically accepted value  $\beta(t)$  as a parameter depending on the time  $t$  of all the procedure.

The aim of the proposed program is a detailed study of the optimal conditions of such a dynamical process with reference to energy and kind of irradiating particles (protons, ions, electrons, gamma quanta), as well as the exposition time, circulation/extraction velocity and a degree of separation. The results of this study will be useful for starting the relevant laboratory test and so, for obtaining information about the possibility of efficient and cheap RW transmutation on a large scale.

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\*A proposal submitted and accepted for realization at the Laboratory of High Energies, JINR, Dubna.

## CURRENT PROBLEMS AND PROSPECTS OF NUCLEAR POWER\*

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The problem of energy production on a large scale for rapidly increasing world population is at the present of vital importance. As a consequence various ways of energy gain are now subjected to thorough and comprehensive analysis from the viewpoint of more and more rigorous and stringent criteria, the main being safe operation, commercial competition, reserves of energy sources and not devastating the earth. So, it becomes apparent that just in not distant future the energy production on a global scale should not consume oxygen and, consequently, not create carbon dioxide which accumulation in ocean waters could lead to catastrophic changes in the ecosystem of our planet which regenerative capabilities are constantly diminish as a result of mankind activity. The unique way that completely satisfies the above-mentioned conditions is only nuclear power (NP). But the future NP should be safe, much more efficient and no producing radioactive wastes.

The increasing population of the world needs more and more energy to satisfy various and constantly rising requirements. Moreover, the prolongation of human life itself strongly depends, in the average, on the energy consumption per head (for example, [1]). Besides, the demand for energy grows faster than the increasing of population because about one half is due to the higher living standards.

The main energy sources currently used are: fossil fuels (gas, oil, coal), fast enough flowing rivers producing the so-called hydroelectricity and the binding energy of heavy nuclei which is the base of nuclear power. However, the total potential of hydroelectric power is estimated as not exceeding about 10% of total world's energy requirement whereas the current share of nuclear power in electricity generation approximated ~15-17%. Contribution of other sources called as renewable (wind, tides, biomass, solar, wave and thermal water and geothermal) amounts to several percents of the total demands. Moreover, although very

preferable and continually improving they are as a rule much more expensive and should be spread over large surfaces to collect a profitable quantity of energy.

So, until now the relevant devices (like windmills, wave generators or solar batteries) are of local application being used mainly in isolated places. The same holds true for the profit from the magnitude of energy which now is simply squandered and can be saved, unless some new physics phenomena will be involved, such as reliable high-temperature superconductors or quite new fascinating and crazy approach to the use of electricity as, for example, by means of low-voltage devices supplied by local, even home, electrochemical sources – fuel cells [2].

The fundamental significance for the world energy policy has the problem of reserves of energy sources. At present it is found reliably enough that fossil fuels (gas and oil) suffice for about 50 years and 200 to 400 years (coal and lignite). Nuclear fuel sources are much richer, both fissile (uranium and thorium) and fusion (deuterium).

Although the traditional kinds of energy production seem to be sufficient, at least for several following decades, nevertheless it is anticipated that between 2008 and 2010 a serious energy gap will arise which could be hardly filled by the renewable energy sources [3]. Moreover, NP is a unique source of energy for spacecrafts and big submarines. It is also assumed to have a wide application in hydrogen production [5]. But the main impediments to the wide present and future use of NP are till now the absence of an extensive agreement about waste management and disposal.

In 2001 there were in operation 439 reactor blocks throughout the world, which produced 16% of electric power [6]. It means that this percentage has been reduced by 4% as compared to the year 1999 although the number of reactor units in service increased by 3 [7]. Nevertheless, in the decade 1988-1998 NP left behind all its main competitors (gas, rock-oil, coal and hydro energy) in

respect of the pace of development and in Europe it held the second place.

Current development of NP is considered as stable and permanent. Its present state is determined by two main factors: a global economic recession involving the apparent surplus of electric energy production (EP) and the transformation of EP market (for example, [6]). But in spite of present difficulties nuclear reactors, in contrast to

the power stations burning oil, gas and coal, are regarded to be, in particular, the most suitable heat source for hydrogen (H<sub>2</sub>) production in a large scale [5].

Nowadays three scenarios of NP development are examined [9]: pessimistic, basic and optimistic. The summary electric capacity of nuclear power stations (NPs) for the period from 2000 to 2020 is shown in Table 1.

Table 1. Total electric capacity of NPs for three scenarios of development of the world NP (in %) with respect to the year 2000 (after [8]).

Years	2000 <sup>*/</sup>	2005	2010	2015	2020
Scenario:					
Pessimistic	100	101,2	100,8	98,2	88,0
Basic	100	103,4	108,0	113,5	115,5
Optimistic	100	106,5	118,1	127,8	138,9

<sup>\*/</sup>Total world electric capacity of NPs in 2000 was 350,6 GW.

It should also be stressed that the above estimations concern the currently functioning NP based mainly on light water thermal reactors.

Present NP is quite competitive as compared to other kinds of energy production on a large scale and sufficiently safe as well. It is worthy mentioning that, for example, in USA the electricity charges for "nuclear" 1 kWh decreased in 80<sup>th</sup> to the lower level than that of the cheapest thermal electric power stations [6]. Nevertheless, in parallel with enormous and till now practically not exhausted potentialities NP has serious shortcomings, in particular, the release of hazardous long-lived and highly toxic radioactive wastes (RW) which could be improperly used. Moreover, in light water reactors that constitute the essential part of present-day NP no more than about 1% of excavated natural uranium is burned up. The problem of passivation of RW remains so far open because their transmutation by means of reasonably available beams of particles (neutrons, relativistic protons and ions) is prohibitively inefficient and expensive, at least as conducted by the traditional way (see, for example, [10]). Indeed, the experiments performed until now in different laboratories show that this problem represents on the practical level a considerable challenge. The point is that when even a precisely isotopically separated sample of a given radioactive nuclide is irradiated in the neutron flux of  $\sim 10^{14} \text{ s}^{-1} \text{ cm}^{-2}$  intensity as the most efficient, then progressively other radioactive nuclides appear and during the first several years of irradiation the total radioactivity of the sample increases. For example, in the case of plutonium incineration the maximum of the sample radioactivity sets in at 8.2 years after the irradiation is started [11]. The situation relative to other candidates for incineration: Am and Cu [11] is even more complicated. Moreover, owing to the intricate structure of neutron-heavy nuclei cross-sections, the efficiency of transmutation/incineration strongly depends on energy spectra of neutrons and this dependence, in turn, changes according

to the conditions of irradiation. At the same time there are known the results about the dependence of the direction of transmutation on the intensity of neutron fluxes [11, 12].

Much more difficult situation occurs if the irradiated sample is not initially strictly determined isotopically but, for example, it is simply taken as a piece of the spent reactor fuel. So, if we take into account that radioactive and, as well, radiotoxic nuclides have, in general, very complex structure of their transmutation it becomes evident that the transmutation/incineration of radio-nuclides in bulk is a very serious and challenging problem which requires comprehensive analysis to bring it in the future to the practical level on the large scale.

During several last years intense investigations of the RW transmutation (RWT) process have been conducted in many laboratories and first quantitative estimations have been obtained of the yield of some processes used for the major long-lived nuclides: <sup>99</sup>Tc, <sup>129</sup>I, <sup>237</sup>Np and <sup>239</sup>Pu occurring in acceptable conditions via the chains of complex enough reactions [15,16].

So, a proposal has been put forward suggesting to use for transmutation a liquid target of RW circulating in the field of high energy and intensity relativistic particles beams when stable and short-living radioactive nuclides [10] are gradually extracted from the exposed target. It should be stressed that the problem of management and disposal of RW is the subject of international research programs on the large scale [17].

Other important problems facing the present-day NP are the low efficiency of uranium fuel used for energy production in thermal reactors.

The existing knowledge in the field of nuclear physics enables to solve the above-mentioned problems in the combined way. So, according to rather common opinion based on quantitative estimations the most prospective arrangements of future NP are subcritical nuclear reactors driven by accelerators producing high-current beams of relativistic protons called the

Accelerator Driven Systems (ADS) [18-21]. Such arrangements are quite safe because the reactors or a system of reactors operate in deeply subcritical regime, i.e. at the effective factor of neutron multiplication  $\sim 0.94-0.98$ , whereas the missing part of neutrons is provided by spallation reactions initiated in heavy and massive target by a beam of relativistic protons from accelerator. Therefore, in order to switch off immediately all this arrangement it is enough to stop the proton fuelling. At the same time the ADS give the possibility to produce the high fluencies ( $\sim 10^{16}$  n/cm<sup>2</sup>s) of neutrons with energy above  $\sim 1$  MeV which are necessary for RWT.

At present, much more prospective seems to be the ADS with cascade composition of two reactors. In this case the fast reactor operating in a hard neutron flux ( $\sim 1$  MeV) is directly fuelled by spallation neutrons but the thermal reactor produces the main part of energy. By means of computer modelling it has been found that such a configuration gives the possibility to minimize the proton flux by one order of magnitude [20]. Moreover, the efficiency of RWT increases noticeably.

Nevertheless, for practical realization of the above-mentioned conceptions it is necessary to solve several important problems, in particular, the optimisation of: fuel isotopic composition and its phase state (solid or liquid), the magnitude of flux and energy of protons from accelerator, isotopic composition of spallation target taking into account its radiation damage and cooling, radiation resistance of material of entrance window between accelerator and reactor core, isotopic composition of moderators and reflectors, and the investigation of radiation processes in materials and in coolants.

In the meantime work in progress on the project of the high-temperature gaseous reactors (Pebble Bed Modular Reactor) with globular fuel elements, which are expected to be used, in particular, for the large-scale production of hydrogen (so-called hydrogen economy) [4,23]. Small nuclear heat sources for space applications are also the subject of wide current discussion [24].

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**RESTORATION OF HEAVY-ION POTENTIALS  
DEDUCED FROM THE GLAUBER-SITENKO MULTIPLE SCATTERING METHOD  
AT INTERMEDIATE ENERGIES**

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Starting from the double-folding potential of two colliding nuclei [1], denoted as 1 and 2,

$$U(r) = \frac{1}{(2\pi)^3} \int d^3q \exp(-i\mathbf{q}\mathbf{r}) \rho_1^o(q) \rho_2^o(q) v_{NN}(q), \quad (1)$$

where  $\rho^o(q)$  is a form factor of the point-like density of a nucleus and expressing the Fourier transform of the NN potential by the NN amplitude as  $v_{NN}(q) = -(E/k) \cdot \sigma_{NN}(i + \alpha_{NN}) \cdot f(q)$ , where  $f(q) = \exp(-q^2 r_{NN}^2/6)$  is the form factor, one obtains

$$U(r) = V(r) + iW(r) = -\frac{2E}{k(2\pi)^2} \sigma_{NN} \quad (2)$$

$$(i + \alpha_{NN}) \int_0^\infty dq q^2 j_0(qr) \rho_1^o(q) \rho_2^o(q) f(q)$$

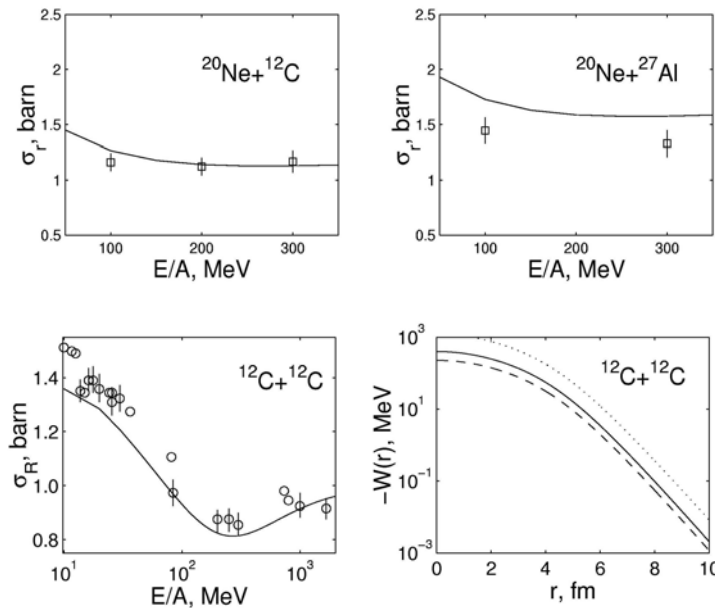
Here  $\sigma_{NN}$  is the total NN-cross-section and  $\alpha_{NN}$  - the ratio of the real to imaginary part of the NN-amplitude. On the other hand, substituting in (1) the equality  $\mathbf{q}_r = \mathbf{q}_\perp \mathbf{b} + \mathbf{q}_\parallel z$  ( $q^2 = q_\parallel^2 + q_\perp^2$ ) in the cylindrical frame, where  $b$  is the impact parameter and  $d^3q = q_\perp dq_\perp d\phi dq_\parallel$ , and using the equality  $\int dq_\parallel \exp(-iq_\parallel z) = 2\pi \delta(z)$ , one can derive at  $q_\perp \rightarrow q$  the phase  $\Phi(b) = -(k/2E) \cdot \int dz U((b^2+z^2)^{1/2})$  as follows

$$\Phi(b) = \frac{1}{4\pi} \sigma_{NN} (i + \alpha_{NN}) \int_0^\infty dq q J_0(qb) \rho_1^o(q) \rho_2^o(q) f(q) \quad (3)$$

This form corresponds to the one derived in the spatial coordinates in [2], basing on the optical-limit model of the Glauber-Sitenko theory [3]. In [4] this phase was used for calculations of the nucleus-nucleus total reaction cross-sections

$$\sigma_r = 2\pi \int_0^\infty db b [1 - \exp(-2 \text{Im} \Phi(b))] . \quad (4)$$

Thus, we conclude that if one obtains an agreement with experimental data by using expressions (3) and (4), then the respective imaginary part of the optical potential can be estimated with the help of (2). In (1)-(3), the form factors of the point-like densities  $\rho^o(q)$  can be expressed by  $\rho(q)$  and  $\rho_0(q)$  for nuclei and a nucleon form factor - by the relation  $\rho(q) = \rho^o(q) \rho_0(q)$ . In [4] it was shown that at low  $q$  one can take  $\rho_0(q) \exp(-q^2 r_0^2/6)$  and put *rms*-radii of the nucleon  $r_0$  and the NN-interaction  $r_{NN}$  to be equal in magnitude. This latter results in the equality  $\rho_2^o(q) f(q) = \rho_2(q)$ , where  $\rho_2(q)$  is known from electron scattering data. From the figures one sees a fairly well agreement of our  $\sigma_r$  calculations with experimental data [5]. We used the realistic shape for density distributions  $\rho(r)$  in the form of the symmetrized Fermi function  $\rho_{SF}(r) = \rho(0) \cdot \sinh(ar) / [\cosh(ar) + \cosh(aR)]$  with parameters taken from [4]. Note that its form factor has the exact form  $\rho_{SF}(q) = -\rho(0)(4\pi a/q)(d/dq)[(\sinh(qR)/\sinh(\pi a q)]$  [6]. The corresponding imaginary parts of the potential (2) for  $^{12}\text{C}+^{12}\text{C}$  at different energies (in MeV)  $E/A_1 = 50$  (solid), 200 (dashed), 2000 (pointed), are shown in the right hand side of the figures. In calculations, we did not adjusted the parameters of  $\rho(q)$ . They were taken from experimental data on nuclear form factors.



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### FLUCTUATION OF ELECTROMAGNETIC CASCADE AXIS IN DENSE AMORPHOUS SEGMENTED MEDIA

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The investigation of fluctuation of the so-called energy cascade axis around the geometry cascade axis for electromagnetic cascade produced in liquid xenon by gamma quanta within the energy interval 100-3500 MeV has been performed. As a basis the previously obtained experimental data [1] from the 180-liter Xenon Bubble Chamber of ITEP (Moscow) [2] were used. Our results may be helpful for the construction and further enhancement of the characteristics of electromagnetic calorimeters with fine-segmented active absorbers such as, for example, PANDA (GSI, Darmstadt).

Although the information, both experimental and numerical, about basic characteristics of electromagnetic (em) cascades (EMCs), mostly integral ones, like average longitudinal and lateral profiles, produced in dense amorphous materials by high enough energy photons and electrons (and positrons) has been collected for many years (for example, [1]), nevertheless our present knowledge of many other practically important features of this process is insufficient till now. This is especially true in regard to fluctuations and correlations of various traits of EMCs. In particular, for the purpose of designing new detectors of hard em radiation, i.e. electromagnetic calorimeters (ECs) based on segmented active absorbers or to improve the characteristics of already existing detectors, of great importance are the knowledge on fluctuations of the so-called energy axis of the cascade (ECA) around its geometric axis (GCA). Note that as distinct from track detectors in ECs the GCA is not determined directly. Therefore the question of estimation of fluctuations of the (experimentally reconstructed) GCA around the ECA (not observed in ECs) is still an important question. For example, an effective mass  $m_{\gamma\gamma}$  of two gamma quanta is of the form:  $m_{\gamma\gamma}^2 = 2(1 - \cos\Theta_{\gamma\gamma})E_{\gamma_1}E_{\gamma_2}$ , where  $E_{\gamma_1}$  and  $E_{\gamma_2}$  are the energy values of each gamma quantum and so the error of the determination of the angle  $\Theta_{\gamma\gamma}$  between them may be as large as several percents or even larger.

Using our data base [1] containing 918 events of EMCs produced in the 180XeBC [2] by gamma quanta of energy from 100 MeV to 3.5 GeV divided into 21 intervals the analysis of ECA fluctuations around their GCA has been performed. The measurements were

made on condition that the used grid has the segmentation cells of dimensions  $\Delta t=0.6$  radiation length (r.l.) along the cascade depth and  $\Delta p=0.3$  r.l. in its lateral direction (see [1], p.189). Notice also that the cut-off energy for electrons and positrons in liquid xenon is  $E_c \approx 1 \div 1.5$  MeV [1]. The experimental data on ECAs were fitted to the following linear dependence

$$p(t | E_{\gamma}) = a(E_{\gamma})t + b(E_{\gamma}) \quad (1)$$

by the least-squares techniques. Here  $p(t | E_{\gamma})$  is the lateral deviation of weighted energy release at the cascade depth  $t$  from geometric axis, whereas the coefficients  $a(E_{\gamma})$  and  $b(E_{\gamma})$  were calculated as a result of the fit to experimental results at all energy values  $E_{\gamma}$ . Note that  $a(E_{\gamma}) = \tan \Theta(E_{\gamma})$  where  $\Theta(E_{\gamma})$  is the average angle between GCA and ECA at the energy  $E_{\gamma}$  and  $b(E_{\gamma})$  is a typical uncertainty of the conversion point determination. The energy dependence of these coefficients is presented in Fig.1. All length parameters:  $p(t | E_{\gamma})$ ,  $t$  and  $b(E_{\gamma})$  are expressed in units of r.l. For liquid xenon 1 r.l.  $\approx 4$  cm [1]. For other absorbers being of interest, for example, for PANDA EC (GSI, Darmstadt) 1 r.l. equals: 1.85 cm for CsI(Tl), 1.68 cm for CeF<sub>3</sub> and 0.89 cm for PbWO<sub>4</sub>. The energy dependence of these parameters can be described by the following simple functions:

$a(E_{\gamma}) = \beta \cdot E_{\gamma}^{-\alpha}$ , where  $\ln \beta = 1.88 \pm 0.47$ ,  $\alpha = 0.74 \pm 0.08$ , and  $b(E_{\gamma}) = (-0.043 \pm 0.008) \cdot \ln E_{\gamma} + (0.50 \pm 0.05)$ ,  $E_{\gamma}$  is in MeV,  $b(E_{\gamma})$  is in r.l.

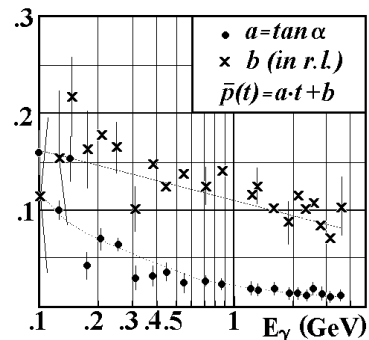


Fig. 1. The fitted parameters  $a = \tan \Theta$  and  $b$  of the energy shower axis. Drawn are the relevant approximation functions.

Fluctuations of coefficients  $a(E_\gamma)$  and  $b(E_\gamma)$  are adequately described by the (non-weighted) root means square (r.m.s.) of these quantities. They are depicted in Fig.2.

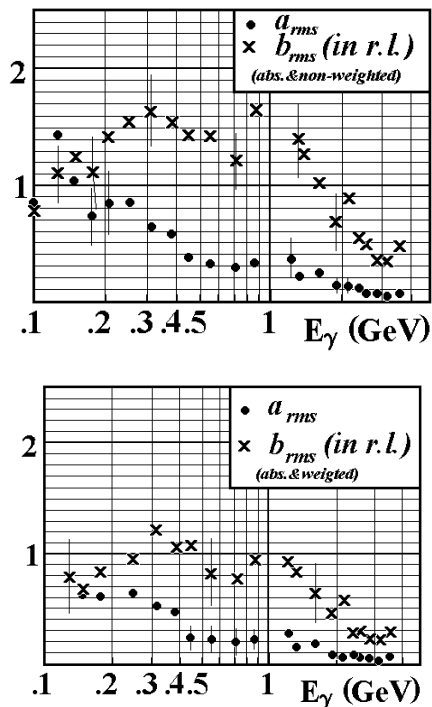


Fig. 2. Energy dependence of r.m.s.-values of the coefficients  $a(E_\gamma)$  and  $b(E_\gamma)$ : non-weighted (left) and weighted (right).

The results obtained in the work can be summarized as follows:

1. The investigated fluctuations of the ECA around the GCA become less than  $\sim 1.5^0$  at  $E_\gamma \approx 500$  MeV and decrease to about  $0.6^0$  at  $E_\gamma \approx 2$  GeV (Fig.1) on condition that the chosen segmentation of active absorber (liquid xenon) is as  $\Delta t \times \Delta p = 0.6 \times 0.3$  r.l.<sup>2</sup>. It means that the accuracy of identification of the

lightest particles decaying into two gammas -  $\pi^0$ -mesons - is about 10% when the accuracy of energy determination is not worse than 20%.

2. At lower energies, i.e. when  $E_\gamma \leq 100$  MeV, these fluctuations rapidly increase because the process of EMC is much less outlined and the notion of ECA loses its practical sense.
3. The real situation is somewhat worse when the GCA is directed not exactly along the longitudinal segmentation of the active absorber.
4. The fluctuations under investigation diminish when the dimensions of pixels are lower and lower, and they increase at larger pixels.
5. The coefficients  $a(E_\gamma)$  and  $b(E_\gamma)$  (and their r.m.s. estimates) depend not only on the pixels dimensions but also on the cut-off energy  $E_c$  in such a way that the fluctuations decrease with the decreasing  $E_c$ . In our approximation of  $a(E_\gamma)$  it means that the parameter  $\alpha$  increases with the decreasing  $E_c$ . The exhaustive qualitative analysis of these dependences is available only with the help of computer EMC process simulation. It can be done, for example, using the EGS4 code [3] and GEANT.

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## ON NUMERICAL SOLUTION OF LARGE SYLVESTER EQUATIONS

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### 1. Problem formulation

The main purpose of this note is to describe a simple iterative method for solving the matrix equation

$$\mathbf{A}\mathbf{X} - \mathbf{X}\mathbf{B} = \mathbf{C}, \quad (1)$$

Where  $\mathbf{A} \in \mathbb{R}^{m \times m}$ ,  $\mathbf{B} \in \mathbb{R}^{n \times n}$  and  $\mathbf{C}, \mathbf{X} \in \mathbb{R}^{m \times n}$ , representing the Sylvester equation which finds many applications in control theory, e.g., for  $\mathbf{B} = -\mathbf{A}^T$  we have the well known Lyapunov equation.

The matrix equation (1) possesses a unique solution if and only if the matrices  $\mathbf{A}$  and  $\mathbf{B}$  have no common eigenvalues, and it can be equivalently written as a large linear system of the following form

$$\mathbf{G}\mathbf{x} = \mathbf{c} \quad (2)$$

with an  $mn \times mn$  matrix

$$\mathbf{G} = \mathbf{I}_n \otimes \mathbf{A} - \mathbf{B}^T \otimes \mathbf{I}_m \quad (3)$$

where  $\otimes$  denotes the Kronecker product, and  $\mathbf{x}, \mathbf{c} \in \mathbb{R}^{mn}$  are vectors whose components are the entries of successive rows of the matrices  $\mathbf{X}$  and  $\mathbf{C}$  respectively.

A nice survey of properties and applications of the Sylvester equation in control theory is presented by Datta and Datta [1]. Recently, among the others, Hu and Reicher [2] proposed new Krylov subspace algorithms based on Arnoldi and Lanczos methods.

In the following section, the simple algorithm of SOR-like method [3] is presented for solving Eq.(1). The results of numerical experiments with the solution of separable elliptic partial differential equations are reported in Section 3 for the examples taken from [2].

**2. The algorithm of SOR-like method**

Assuming that the matrix A is defined by the following decomposition:

$$\mathbf{A}=\mathbf{K}-\mathbf{L}-\mathbf{U}, \tag{4}$$

where **K**, **L** and **U** are nonsingular diagonal, strictly lower triangular and strictly upper triangular parts of **A**; Eq.(1) can be rewritten as

$$\mathbf{KX} = \mathbf{LX} + \mathbf{UX} + \mathbf{XB} + \mathbf{C} \tag{5}$$

or equivalently

$$\mathbf{X} = \mathbf{K}^{-1}\{\mathbf{LX} + \mathbf{UX} + \mathbf{XB} + \mathbf{C}\}. \tag{6}$$

The iterative process can be written in the form

$$\mathbf{X}^{(t)} = \mathbf{K}^{-1}\{\mathbf{LX}^{(t)} + \mathbf{UX}^{(t-1)} + \mathbf{X}^{(t-1)}\mathbf{B} + \mathbf{C}\} \tag{7}$$

for  $t = 1, 2, \dots$ ,

where for computing the product **XB**, the entries of  $\mathbf{X}^{(t-1)}$  and  $\mathbf{X}^{(t)}$  are used.

For the acceleration of convergence in the above scheme, the overrelaxation procedure can be used as follows

$$\mathbf{X}^{(t)} = \omega\mathbf{K}^{-1}\{\mathbf{LX}^{(t)} + \mathbf{UX}^{(t-1)} + \mathbf{X}^{(t-1)}\mathbf{B} + \mathbf{C}\} - (\omega - 1)\mathbf{X}^{(t-1)}. \tag{8}$$

for  $t = 1, 2, \dots$ ,

or written equivalently

$$\mathbf{X}^{(t)} = [\mathbf{I} - \omega\mathbf{K}^{-1}\mathbf{L}]^{-1}\{[(1 - \omega)\mathbf{I} + \omega\mathbf{K}^{-1}\mathbf{U}]\mathbf{X}^{(t-1)} + \omega\mathbf{K}^{-1}[\mathbf{X}^{(t-1)}\mathbf{B} + \mathbf{C}]\} \tag{9}$$

for  $t = 1, 2, \dots$

As is observed in numerical experiments the minimum number of iteration occurs usually for  $0 < \omega < 1$  and the method is convergent for this interval of  $\omega$ 's.

In the analysis of the reliability of iterative solutions, it is convenient to consider the (true) error matrix

$$\mathbf{E}^{(t)} = \mathbf{X} - \mathbf{X}^{(t)}, \tag{10}$$

the 'inner (or pseudo-residual) error matrix

$$\delta^{(t)} = \mathbf{X}^{(t)} - \mathbf{X}^{(t-1)}, \tag{11}$$

the relative inner error matrix computed entry-wise

$$\bar{\delta}_{j,k}^{(t)} = \frac{x_{j,k}^{(t)} - x_{j,k}^{(t-1)}}{x_{j,k}^{(t)}} \tag{12}$$

and the residual matrix

$$\mathbf{R}^{(t)} = \mathbf{AX}^{(t)} - \mathbf{X}^{(t)}\mathbf{B} - \mathbf{C}, \tag{13}$$

where **X** is assumed as the "exact" solution matrix.

Since the above quantities are matrices, the Frobenius norm is an important matrix norm, for us

$$\|A\|_F = \left[ \sum_{j=1}^m \sum_{k=1}^n |a_{j,k}|^2 \right]^{1/2} \tag{14}$$

and for needs of comparison of obtained results with those given in [2] we redefine the maximum matrix norm as follows

$$\|A\|_{\infty}^* = \max_{\substack{1 \leq j \leq m \\ 1 \leq k \leq n}} |a_{j,k}| \tag{15}$$

Most recent iterative methods terminate when the residual  $\mathbf{R}^{(t)}$  is sufficiently small and the termination test

$$\frac{\|\mathbf{R}^{(t)}\|_F}{\|\mathbf{R}^{(0)}\|_F} \leq \varepsilon, \tag{16}$$

named usually as a relative residual norm, is most commonly used criterion in Krylov subspace algorithms. However, it seems that in the case of iteration methods based on a matrix splitting, the termination test

$$\|\bar{\delta}\|_{\infty}^* \leq \varepsilon \tag{17}$$

can be practically considered as the most useful stopping criterion independent on an used initial guess.

**3. Numerical experiments**

We shall illustrate the numerical performance of the iterative scheme called *SOR-like method* in some examples taken from [2].

Let  $\Omega := \{(x, y) \in R^2 : 0 < x < 1, 0 < y < 1\}$ , and let  $\partial\Omega$  denotes the boundary of  $\Omega$ . We shall solve the following boundary-value problem for the two-dimensional separable model of convection-diffusion equation

$$\left. \begin{aligned} -\Delta u + 2p_1u_x + 2p_2u_y - 2p_3u &= F \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned} \right\} \tag{18}$$

The parameters  $p_1, p_2$  and  $p_3$  are nonnegative in all considered examples. The right-hand-side function  $F(x,y)$  is chosen so that  $u(x,y) = xe^{xy} \sin\pi x \sin\pi y$  solves (18). The Laplacian in (18) is discretized by the standard five-point formula and the first-order derivatives by centered finite differences. The mesh size in both the  $x$ - and  $y$ -directions is  $h = (n + 1)^{-1}$ .

Define the mesh points  $x_j := jh$  and  $y_k := kh$  for  $0 \leq j, k \leq n$ . We seek to determine values of the solution  $\tilde{u}(x,y)$  at interior mesh points  $\{(x_j, y_k)\}_{j,k=1}^n$ . This yields a linear system of algebraic equations that can be written as a Sylvester equation (1) with  $n \times n$  matrices

$$A := \frac{1}{h^2} \text{tridiag}\{-1 - p_1h, 2 - p_3h^2, -1 + p_1h\} \tag{19}$$

$$B := -\frac{1}{h^2} \text{tridiag}\{-1 + p_2h, 2 - p_3h^2, -1 - p_2h\} \tag{20}$$

and  $C = \{c_{j,k}\}_{j,k=1}^n$  is given by

$$c_{j,k} := F(x_j, y_k) \quad (21)$$

Thus, the entry  $(j, k)$  of the solution matrix  $\tilde{U} \equiv X$  of the Sylvester equation approximates  $\tilde{u}(x_j, y_k)$ .

The behavior of the following norms

$$B = \frac{\|R^{(t)}\|_F}{\|R^{(0)}\|_F}, \quad C = \|R^{(t)}\|_F,$$

$$D = \|E^{(t)}\|_F \quad \text{and} \quad E = \|E^{(t)}\|_\infty^*$$

versus the number of iterations is examined for three examples with using the stopping criterion

$$A = \|\bar{\delta}\|_\infty^* \leq \varepsilon + 10^{-12} \quad (22)$$

the zero initial guess and the optimum value of  $\omega$  determined experimentally and computations were performed in double-precision.

In [2,3] the following examples are considered.

EXAMPLE 3.1.

$p_1=p_2=p_3=0$  and  $n = 31$ . Since  $\mathbf{B} = -\mathbf{A}$  and  $\mathbf{A} = \mathbf{A}^T$ , the related Sylvester is equivalent to the Lyapunov equation. The solution is obtained after 195 iterations with  $\omega = 0.915$  and the stopping test (22), where the residual norm  $C$  is satisfied with about  $10^{-11}$  but the remaining norms are satisfied with the values about  $10^{-14}$ .

EXAMPLE 3.2.

$p_1 = 25, p_2 = 50, p_3 = 50$  and  $n = 31$  and both matrices  $\mathbf{A}$  and  $\mathbf{B}$  are nonsymmetric. The solution is obtained after 34 iterations with  $\omega = 0.45$  and the stopping test (22). The residual norm  $C$  is satisfied with about  $10^{-10}$  but the remaining norms are again satisfied with the values about  $10^{-14}$ .

EXAMPLE 3.3.

$p_1 = 50, p_2 = 100, p_3 = 50$  and  $n = 63$ . Both nonsymmetric matrices  $\mathbf{A}$  and  $\mathbf{B}$  have a larger order

than those in the previous examples. The obtained results are similar to those for Example 3.2. From the results of these test computations, it can be concluded that the proposed SOR-like method, represented by the iterative scheme (8), is a very efficient technique for solving Sylvester equations especially with the nonsymmetric matrices  $\mathbf{A}$  and  $\mathbf{B}$ . For considered examples, the method provides much more accurate solutions, obtained with a computational work lesser than a few orders in comparison to solutions obtained by means of Krylov subspace algorithms and given in [2]. The computational work in one iteration of the scheme (8) is roughly equivalent to that required for computing the residual matrix (13). Thus, with using the stopping test (22), the solutions were obtained with the relative residual norm  $B$ , the true error norms

$$D = \|E^{(t)}\|_F \quad \text{and} \quad E = \|E^{(t)}\|_\infty^* \quad \text{lesser than } 10^{-13}; \quad \text{and}$$

with the computational work equivalent roughly to 195, 33 and 38 computations of the residual matrix (13) for Examples 3.1, 3.2 and 3.3 respectively.

For Krylov algorithms analyzed in [6], for example, the solution in Example 3.3 could be obtained only with

$$\text{the true error norm } E = \|E^{(t)}\|_\infty^* \approx 10^{-3}, \quad \text{that is, about}$$

ten orders greater than in the case of the SOR-like method.

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